

A Picard–Brauer Object for Derived Smooth Manifolds

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Part I: Historical Background and Motivation

Geometric setting	Classical Brauer theory	Derived Brauer theory
(Derived) schemes	Grothendieck (1968)	Toën (2012)
topological spaces or (Derived) manifolds	Dixmier–Douady (1963) Donovan–Karoubi (1970) Murray (1996)	?

What should go into the lower-right corner?

The Question

Toën's theory is formulated in the algebraic setting, using the étale topology.

Before asking for derived Azumaya algebras in smooth geometry, we ask a more basic question:

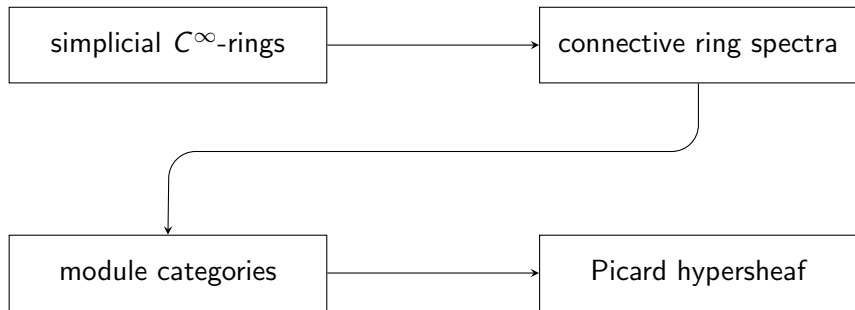
**What is the Picard–Brauer object
of a derived smooth manifold?**

We work with derived smooth manifolds in the sense of Spivak.

From Smooth Geometry to Spectral Geometry

The structure sheaf of a derived smooth manifold is built from simplicial C^∞ -rings.

Picard theory, however, is naturally stable and spectral.



The Key Local Computation

For every point x ,

$$\mathbb{O}_{X,x}$$

is a **connective local ring spectrum**.

By a theorem of Antieau–Gepner, every invertible $\mathbb{O}_{X,x}$ -module is equivalent to $L \simeq \Sigma^n \mathbb{O}_{X,x}$ for a unique integer n . Here $\mathbb{O}_{X,x}$ is the tensor unit.

The automorphism space of the tensor unit is $\mathrm{GL}_1(\mathbb{O}_{X,x})$.

We prove that

$$(\mathrm{Pic}_{\mathbb{O}_X})_x \simeq \mathrm{Pic}(\mathbb{O}_{X,x}),$$

so the local computation globalizes.

Main result and classical comparison

After delooping, we get our main result

$$\mathrm{Br}_X^{\mathrm{Pic}} = B \mathrm{Pic}_{\mathbb{O}_X} \simeq K(\mathbb{Z}, 1) \times B^2 \mathrm{GL}_1(\mathbb{O}_X)$$

Fix the same ordinary paracompact smooth manifold M .

Smooth functions on M	Our open-site Picard-torsor classes	Classical comparison
$C_M^\infty(\mathbb{R})$	$H^1(M; \mathbb{Z}) \times H^2(M; \mathbb{Z}/2)$	Graded: $H^0(M; \mathbb{Z}/8) \times H^1(M; \mathbb{Z}/2) \times H^2(M; \mathbb{Z}/2)$
$C_M^\infty(\mathbb{C})$	$H^1(M; \mathbb{Z}) \times H^3(M; \mathbb{Z})$	Graded: $H^0(M; \mathbb{Z}/2) \times H^1(M; \mathbb{Z}/2) \times H^3(M; \mathbb{Z})_{\mathrm{tors}}$ Finite-dimensional Azumaya algebra: $H^3(M, \mathbb{Z})_{\mathrm{tors}}$ Dixmier-Douady-class: $H^3(M, \mathbb{Z})$

More details are presented on my poster.

Thank you