A Picard–Brauer Object for Derived Smooth Manifolds

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Part I: Historical Background and Motivation

Geometric setting	Classical Brauer theory	Derived Brauer theory
(Derived) schemes	Grothendieck (1968)	Toën (2012)
topological spaces or (Derived) manifolds	Dixmier–Douady (1963) Donovan–Karoubi (1970) Murray (1996)	?

What should go into the lower-right corner?

The Question

Toën's theory is formulated in the algebraic setting, using the étale topology.

Before asking for derived Azumaya algebras in smooth geometry, we ask a more basic question:

What is the Picard–Brauer object of a derived smooth manifold?

We work with derived smooth manifolds in the sense of Spivak.

From Smooth Geometry to Spectral Geometry

The structure sheaf of a derived smooth manifold is built from simplicial C^∞-rings.

Picard theory, however, is naturally stable and spectral.

$$\begin{tikzcd}
\boxed{\text{simplicial } C^\infty\text{-rings}} \arrow[r] & \boxed{\text{connective ring spectra}} \arrow[dl] \\
\boxed{\text{module categories}} \arrow[r] & \boxed{\text{Picard hypersheaf}}
\end{tikzcd}$$

The Key Local Computation

For every point x,

$$\mathbb{O}_{X,x}$$

is a **connective local ring spectrum**.

By a theorem of Antieau–Gepner, every invertible $\mathbb{O}_{X,x}$-module is equivalent to $L \simeq \Sigma^n \mathbb{O}_{X,x}$ for a unique integer n. Here $\mathbb{O}_{X,x}$ is the tensor unit.

The automorphism space of the tensor unit is $\mathrm{GL}_1(\mathbb{O}_{X,x})$.

We prove that

$$\left(\mathrm{Pic}_{\mathbb{O}_X}\right)_x \simeq \mathrm{Pic}\left(\mathbb{O}_{X,x}\right),$$

so the local computation globalizes.

Main result and classical comparison

After delooping, we get our main result

$$\boxed{\mathrm{Br}_X^{\mathrm{Pic}} = B\,\mathrm{Pic}_{\mathbb{O}_X} \simeq K(\underline{\mathbb{Z}},1) \times B^2\,\mathrm{GL}_1(\mathbb{O}_X)}$$

Fix the same ordinary paracompact smooth manifold M.

Smooth functions on M	**Our open-site Picard-torsor classes**	**Classical comparison**
$C^\infty_M(\mathbb{R})$	$H^1(M;\mathbb{Z}) \times H^2(M;\mathbb{Z}/2)$	Graded: $H^0(M;\mathbb{Z}/8) \times H^1(M;\mathbb{Z}/2) \times H^2(M;\mathbb{Z}/2)$
$C^\infty_M(\mathbb{C})$	$H^1(M;\mathbb{Z}) \times H^3(M;\mathbb{Z})$	Graded: $H^0(M;\mathbb{Z}/2) \times H^1(M;\mathbb{Z}/2) \times H^3(M;\mathbb{Z})_{\mathrm{tors}}$ Finite-dimensional Azumaya algebra: $H^3(M,\mathbb{Z})_{\mathrm{tors}}$ Dixmier-Douady-class: $H^3(M,\mathbb{Z})$

More details are presented on my poster.

Thank you